Philosophy 211
Sample In-Class Exam 2
ANSWERS

I. Complete these proofs. There are no additional assumptions. (14)

A. ∃xPx → ∃xQx, ∀x~(Qx & Sx)
⊢ ∀x(Px → ∃y~Sy)

1	(1) ∃xPx → ∃xQx	A
2	(2) ∀x~(Qx & Sx)	A
3	(3) Pa	A
3	(4) ∃xPx	3 ∃I
1,3	(5) ∃xQx	1,4 →E
6	(6) Qb	A
2	(7) ~(Qb & Sb)	2 ∀E
2	(8) ~Qb v ~Sb	7 DEM
2,6	(9) ~Sb	6,8 vE
2,6	(10) ∃y~Sy	9 ∃I
1,2,3	(11) ∃y~Sy	5,10 ∃E(6)
1,2	(12) Pa → ∃y~Sy	11 →I(3)
1,2	(13) ∀x(Px → ∃y~Sy)	12 ∀I

B. ∃y∀xRxy, ∃x∀yRxy,
⊢ ∀z∃x∃y(Rxz & Rzy)

1	(1) ∃y∀xRxy	A
2	(2) ∃x∀yRxy	A
3	(3) ∀xRxa	A
4	(4) ∀yRby	A
3	(5) Rca	3 ∀E
4	(6) Rbc	4 ∀E
3,4	(7) Rbc & Rca	5,6 &I
3,4	(8) ∃y(Rbc & Rcy)	7 ∃I
3,4	(9) ∃x∃y(Rxc & Rcy)	8 ∃I
2,3	(10) ∃x∃y(Rxc & Rcy)	2,9∃E(4)
1,2	(11) ∃x∃y(Rxc & Rcy)	1,10∃E(3)
1,2	(12) ∀z∃x∃y(Rxz & Rzy)	11∀I

II. Find the errors in each of these proofs and explain why they are errors. (10)

A. ∀x∃y(Px → Qy) ⊢ ∀xPx → ∀yQy

ANS: Step 7 is illegal. The constant ‘b’ occurs in line 4 which line 6 depends on.

1	(1) ∀x∃y(Px → Qy)	A
2	(2) ∀xPx	A
1	(3) ∃y(Pa → Qy)	1 ∀E
4	(4) Pa → Qb	A
2	(5) Pa	2 ∀E
2,4	(6) Qb	4,5 →E
2,4	(7) ∀yQy	6 ∀I
1,2	(8) ∀yQy	3,7 ∃E(4)
1	(9) ∀xPx → ∀yQy	8 →I(2)

B. $\forall x(Dx \rightarrow Ax) \vdash \forall x(\exists y(Hxy \,\&\, Dy) \rightarrow \exists z(Hxz \,\&\, Az))$

1	(1) $\forall x(Dx \rightarrow Ax)$	A
2	(2) $Hab \,\&\, Db$	A
2	(3) Hab	2 &E
2	(4) Db	2 &E
1	(5) $Db \rightarrow Ab$	3 &E
1,2	(6) Ab	4 &E
1,2	(7) $Hab \,\&\, Ab$	3,6 &I
1,2	(8) $\exists z(Haz \,\&\, Az)$	7 ∃I
1	(9) $(Hab \,\&\, Db) \rightarrow \exists z(Haz \,\&\, Az)$	8→I(2)
1	(10) $\exists y(Hay \,\&\, Dy) \rightarrow \exists z(Haz \,\&\, Az)$	9∃I
1	(11) $\forall x(\exists y(Hxy \,\&\, Dy) \rightarrow \exists z(Hxz \,\&\, Az))$	10 ∀I

ANS: Step 10 is illegal. ∃I is the rule used, but the main connective of line 10 is still the →

III. Paraphrase the following English sentences into Predicate Logic using the following translation scheme: (25)

$A\alpha$: α is on Team A
$B\alpha$: α is on Team B
$D\alpha\beta$: α defeated β
m: Mary
t: Tom

1. Tom did not defeat everyone on Team A who was defeated by Mary.

$$\sim\forall x((Ax \,\&\, Dmx) \rightarrow Dtx)$$

2. If no one on Team A defeated Tom, then there is someone on Team A who did not defeat Mary.

$$\sim\exists x(Ax \,\&\, Dxt) \rightarrow \exists x(Ax \,\&\, \sim Dxm)$$

3. Every member of Team A who defeated Mary was defeated by at least one member of Team B. (NOTE: I intend this to allow that different members of Team A who defeated Mary may have been defeated by different members of Team B.)

$$\forall x((Ax \,\&\, Dxm) \rightarrow \exists y(By \,\&\, Dyx))$$

4. If anyone on Team A defeated anyone on Team B, then everyone on Team B was defeated by Mary.

$$\exists x(Ax \,\&\, \exists y(By \,\&\, Dxy)) \rightarrow \forall x(Bx \rightarrow Dmx)$$

5. Tom defeated at most one member of Team A.

$$\sim\exists x\exists y(x \neq y \,\&\, Ax \,\&\, Ay \,\&\, Dtx \,\&\, Dty)$$

III. Determine whether these sentences are true on the given diagrams: (16)

	D1	D2	D3	D4
∀x(Mx → ∃y(Ty & Ayx))	T	T	T	F
∃x(Mx & ∀y(Ty → Ayx))	F	T	F	F
∃x(Tx & ∀y(My → ~Axy))	F	F	T	F
∀x(Tx → ∃y∃z(My & Mz & y ≠ z & Axy & Axz))	F	T	F	F

Diagram 1

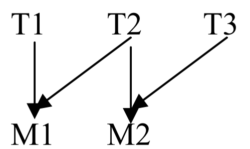


Diagram 2

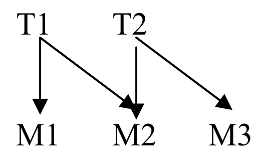


Diagram 3

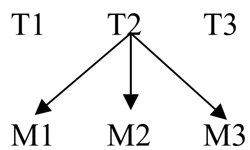
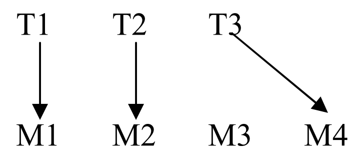


Diagram 4



Prove these sequents: (35)

∀x∀y(~Rxy → Ryx), ∀x∀y(Rxy → Ryx) ├ ∀x∀yRxy
∀x(Px → ~Qx), ∃x(Sx & Qx) ├ ∃x(Sx & ~Px)
∃x∀yRxy, ∀x∀y(Rxy → ~Ryx) ├ ∀x∃y~Rxy
∃x∀yRxy, ∀x((Px & ∃yRyy) → ∀y~Ryx) ├ ∀x~Px

1	1) ∀x∀y(~Rxy → Ryx)	A
2	2) ∀x∀y(Rxy → Ryx)	A
1	3) ~Rab → Rba	1 ∀E x2
2	4) Rab → Rba	2 ∀E x2
1,2	5) Rba	3,4 Spe Dil
1,2	6) ∀yRby	5 ∀I
1,2	7) ∀x∀yRxy	6 ∀I

1	1) $\forall x(Px \rightarrow \sim Qx)$	A
2	2) $\exists x(Sx \,\&\, Qx)$	A
3	3) $Sa \,\&\, Qa$	A [for $\exists$E]
3	4) Sa	3 &E
3	5) Qa	3 &E
1	6) $Pa \rightarrow \sim Qa$	1 $\forall$E
1,3	7) $\sim Pa$	5,6 MT
1,3	8) $Sa \,\&\, \sim Pa$	4,7 &I
1,3	9) $\exists x(Sx \,\&\, \sim Px)$	8 $\exists$I
1,2	10) $\exists x(Sx \,\&\, \sim Px)$	2,9 $\exists$E(3)

1	1) $\exists x \forall y Rxy$	A
2	2) $\forall x \forall y(Rxy \rightarrow \sim Ryx)$	A
3	3) $\forall y Ray$	A [for $\exists$E]
3	4) Rab	3 $\forall$E
2	5) $Rab \rightarrow \sim Rba$	2 $\forall$E x2
2,3	6) $\sim Rba$	4,5 $\rightarrow$E
2,3	7) $\exists y \sim Rby$	6 $\exists$I
2,3	8) $\forall x \exists y \sim Rxy$	7 $\forall$I
1,2	9) $\forall x \exists y \sim Rxy$	1,8 $\exists$E(3)

1	1) $\exists x \forall y Rxy$	A
2	2) $\forall x((Px \,\&\, \exists y Ryy) \rightarrow \forall y \sim Ryx)$	A
3	3) $\forall y Ray$	A [for $\exists$E]
4	4) Pb	A [for RAA]
2	5) $(Pb \,\&\, \exists y Ryy) \rightarrow \forall y \sim Ryb$	2 $\forall$E
3	6) Raa	3 $\forall$E
3	7) $\exists y Ryy$	6 $\exists$I
3,4	8) $Pb \,\&\, \exists y Ryy$	4,7 &I
2,3,4	9) $\forall y \sim Ryb$	5,8 $\rightarrow$E
2,3,4	10) $\sim Rab$	9 $\forall$E
3	11) Rab	3 $\forall$E
2,3	12) $\sim Pb$	10,11 RAA(4)
2,3	13) $\forall x \sim Px$	12 $\forall$I
1,2	14) $\forall x \sim Px$	1,13 $\exists$E(3)